Environmental policy : an evolutionary perspective

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"Environmental Policy: An Evolutionary Perspective"

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Abstract

The environmental issue is usually addressed from a cost on abatement / preservation perspective. But to effectively conserve the environment, we have to be actually willing to do so, which might not always be the case. In the present work, I address the polluting problem from a 'political' point of view. Is it possible to reach a sufficient amount of people who are actually willing to sustain a good environment quality? Or will we always be in a state dominated by the less environmentally aware people? With the help of a simple model, we will find that the result depends, amongst others, on the initial state and on some possible psychological aspects.

Contents

1 Introduction

The present thesis has been motivated by the fact that, sometimes, long term objectives, which undoubtedly have social benefits, are not always reached. Worse than that, the course of action may not always pursue these objectives at all. So the question that arises is: are we, as a society, able to reach said objectives, bearing in mind that we live in a political framed society, where decisions are taken by a ruling party. In this case, I am actually talking about a government, elected in a democratic way. Therefore, one could expect that this government will actually render decisions according to this objective. Reality has proven that these premises are not always fulfilled.

Following this idea and getting into a more specific topic, I would like to address the environmental problem. It seems quite clear that enjoying a good and sustainable environment would be beneficial to everybody and therefore governments should actually act accordingly. But preserving the environment also has its costs. And not everybody is willing to pay these costs. This might have different causes. Different people can evaluate the quality of the environment in different ways. Different people might have different preferences between environmental quality and, for example, their own consumption. Poorer people will prefer to have food and housing first, despite of the quality of the environment (up to some extent). Therefore, we are faced with some heterogeneity in the position of people regarding the environment and how the topic should be addressed.

This might explain, at least partially, why some countries act in one way and others in another. Questions like: 'why did Europe singe the Kyoto Protocol when the USA did not?' could actually be related to some cultural features. How can there prevail different environmental situations in countries which are similar from both an economic and a development perspective?s. It also might be the explanation of why, in the same country or society, some people are more environmentally aware than others. It might the case that some other country specific characteristic could impact on this heterogeneity, like, for example, their income or Gini index.

Therefore, the topic that this dissertation wants to address is: can a long term objective, like a good environment quality, be 'socially' or 'politically' reached? Do we have enough 'green' people in our society in order to actually reach a 'green' equilibrium? How does the interplay between the utility of goods consumption and the disutility of the pollution evolves? Can these 'green' preferences win over the classical pure homo-economicus preferences in the future generations? Will our children be more or less sensible to the environmental status?

It seems that, so far, the environmental issue has been addressed only in such a perspective: the pollution and the environment. Which limits can be imposed on polluting? Are there any incentives that can be put in place so as to abate it? Multiples papers on the matter usually approach this issue from that perspective. Best abatement policies. Information on the products we buy, and the like. However, could 'political' forces actually deter the environment quality?

In order to try to answer some of these questions, I have built a simple model in which we have two types of people. The 'green' ones, who are environmentally sensitive. And the 'brown' ones, which are less aware and more concerned by direct economic profits. This will shape our 'society', by electing a government according to a simple majority rule. Depending on the type of government elected, some incentives will be put in place to protect the environment and therefore the evolution of it will be different according to this. At the same time, we will have that these two traits, the 'green' and 'brown' characteristics, will be passed on from generation to generation. In order to model this feature, I will use the ideas addressed in Bisin and Verdier (2000)[3], Brekke et al.(2003)[4] and Nyborg et al.(2003)[10]. In these models, the idea is that a specific trait is passed on to the following generation with a given probability, which might depend on different things (depending on the model). In the case of the present work, the evolution of the society will be shaped according to the replicator dynamics as in Taylor & L. Jonker (1978) [12].

This will give us a evolution motion equation and will allow us to find different equilibria point, according to some exogenous conditions (parameters) and also based on the initial conditions of the society. Finally, I will obtain some insights on how this type of model can evolve and its implications.

The present dissertation is organized as follows: Section 2 introduces the model and how brown and green people behave. Section 3 deals with the model of cultural transmission. Section 4 explains how the equilibrium point q^* is reached. Section 5 explains how a green government behaves (green policy), leading us to section 6, where the equilibria points are addressed and explained, as well as their welfare consequences. Section 7 explains the existence of a 'pollution trap'. Finally, Section 8 concludes.

2 The People

2.1 The Agent

As stated in the introduction, the idea here is to study the possible 'behaviour' of a society according to a 'green' and 'brown' people distribution and preferences. Therefore, in the present model, we will have a society with a constant population (normalized to 1) which will be compound of a share of 'green' people q and its consequent share of 'brown' people $(1-q)$.

This society can consume two types of goods, which I will call the 'green good' (c_t) and 'brown good' (d_t) - abusing a little with the nomenclature. The difference between these goods is that the green one does not pollute, whereas the brown one does. With this, we will have that the level of pollution P_t will depend on the amount of brown good consumed. The relationship will be:

$P_t = \gamma d_t$[1]. The 'drawback' of the green product is that it is more expensive than its brown counterpart. In this model, I will assign a normalized price of 1 to the brown good and a price of $(1+\rho)$ to the green one. Therefore the value of ρ represents the extra amount (with respect to the whole original price) to be paid to buy a green product.

Finally, before going into the details of the model, I will assume that each agent has a fixed wage of w, which they will use to consume these goods. The share of each good will be discussed in the following section.

In this model, the difference between the green and brown agents is due to the first one having a 'moral gain' of being a responsible and environmentally friendly person. Therefore, this agent will be willing to 'contribute' to the environment by buying green products, even thought these are more expensive. I will call 'r' the share of green goods that green people buy.

Accordingly, I will define the *green willingness* = δ, which can be interpreted as the percentage (in goods) that the green person is willing to give away in order to contribute to the environment. It can be also be thought as the percentage of net income given to the green cause (both are equivalent). It will be assumed that δ is fixed.

Therefore the utilities functions can be written as follows:

$$U_{brown}(n_{1t}, P_{t-1}) = f(n_{1t}) - h(P_{t-1}) \tag{2.1}$$

$$U_{green}(n_{2t}, P_{t-1}) = f(n_{2t}) - h(P_{t-1}) + m(r, P_t) \tag{2.2}$$

where:

Variable	**Meaning**
η	Environment "self" restoring.
γ	Impact of brown (total) consumption on the environment.
ρ	"Mark up" for green products (with respect to brown ones).
P_{t-1}	Pollution level in period $(t-1)$, which is observed at the beginning of period t.
n_{1t}	Total amount of consumed goods (brown and green products) by a **brown** agent. Subsequently it will be just called n_1.
n_{2t}	Total amount of consumed goods (brown and green products) by a **green** agent. Subsequently it will be just called n_2.
$f(n)$	It is the economic part of the utility function. As usual, it will be an increasing function, with diminishing marginal returns.
$h(P_t)$	It is the disutility of pollution. It is increasing function with $\frac{\partial h}{\partial P_t} > 0$.
$m(r, P_t)$	Moral utility (only for green people) which is increasing with r and P_t.

[1] A more 'realistic' evolution equation could be used, for example: $P_t = (1-\eta)P_{t-1} + \gamma d_t$.

2.2 The brown people

Since at the beginning of period t the pollution level is given, the brown people will only maximize the economic part of their utility function, subject to the budget constraint, which in this case is simply the wage w. Therefore their consumption n_1 will be governed by the simple identity $n_1 = w$.

2.3 The green people

In the case of the green people, they buy a share 'r' of green products and the rest $(1-r)$ in brown products. Since the total amount of product bought by a green person is n_2, then he/she will buy $r \cdot n_2$ green products (at a price of $(1+\rho)$) and $(1-r)n_2$ brown products (at price equal to 1). Therefore their budget constraint is:

$$(1+\rho)r \cdot n_2 + (1-r)n_2 = w$$

which becomes:

$$n_2(1+r\rho) = w \tag{2.3}$$

or

$$n_2 = \frac{w}{(1+r\rho)} \tag{2.4}$$

Following the '*green willingness*' idea, as defined before, this is the proportion of products (in quantity) that the green person is willing to 'give up' in order to be green, in comparison of just behaving as a brown person. Consequently, the equations are:

$$\begin{aligned}
\frac{(n_1 - n_2)}{n_1} &= \delta \\
1 - n_2/n_1 &= \delta \\
n_2/n_1 &= 1 - \delta \\
\frac{1}{(1+r\rho)} &= (1-\delta) \\
r &= \frac{\delta}{(1-\delta)\rho}
\end{aligned}$$

And, therefore, the fraction r is determined by the green willingness (δ) and the extra price of the green goods (ρ). Obviously, this fraction is bounded by 1, and therefore the final expression for r is:

$$r = \min\left(1, \frac{\delta}{(1-\delta)\rho}\right) \tag{2.5}$$

2.4 The moral gain

I will use the moral gain or moral satisfaction as it is addressed in Nyborg et al.(2003) [10]. In the present model, I will suppose that green people gain their moral satisfaction from contributing in preserving a good environment and/or improving the environmental quality. Therefore, their gain will come from their contribution, which in our case would be represented by r. It will also be a increasing function with respect to the environment P_t.[2]

Also, I will suppose that this moral gain is a comparative matter, meaning that the green person's gain comes from the comparison they do with respect to a mean person contribution, in a similar fashion as stated in 'Catching Up with the Joneses', Alvarez et al.(2004) [1]. Therefore, their moral satisfaction will be an increasing function of $(r - \bar{r})$. Acknowledging that $\bar{r} = r \cdot q + 0 \cdot (1 - q) = r \cdot q$ and that $m(\cdot)$ is an increasing function in both arguments, we have:

$$\begin{aligned} \textit{moral gain} &= m((r - \bar{r}), P_t) \\ &= m((r - rq), P_t) \\ &= m(r(1 - q), P_t) \end{aligned}$$

In our model we will use, for simplicity in the following calculations, the subsequent function:

$$m(r(1 - q), P_t) = \alpha r(1 - q)P_t$$

Now, recalling what was seen at the beginning of this section, we have that $P_t = \gamma \cdot d_t$, being d_t the (total) brown good consumption. A simple computation yields to[3]: $d_t = (1 - r)q + (1 - q) = (1 - rq)$. Replacing this in the previous equation, we get:

$$m(r(1 - q), P_t) = \alpha r(1 - q)\gamma(1 - rq)$$

and rearranging:

$$m(r, q) = \alpha\gamma r(1 - q)(1 - rq) \tag{2.6}$$

where α is just the weight of the individual to this moral satisfaction, in his/her utility function.

Note about the 'real' mean person perception:

I have assumed so far that the green person can actually perceive with perfect precision the real

[2]We can choose a P_{t-1} instead of P_t. The main difference between them, is that in the case with P_{t-1} we will have a two state variable system (q, P_{t-1}), whereas in the other case, $P_t = f(q)$ and, therefore, the system reduces to only one variable: q.

[3]For simplicity, it is assumed that $w = 1$.

mean value of r: $\bar{r}$. Lets now suppose that this perception is biased[4]. Lets call Ω (with $0 < \Omega < \infty$) the distorting factor of perception. In this case, the previous equation will become:

$$m(r, q) = \alpha\gamma r(1-q)(1-\Omega r q)$$

We will analyze this bias effect in section 3, where we will see the impact of Ω in the equilibrium point.

2.5 The cost of being green

Now I will address how the green willingness translates into the cost that the green people assume, in utility units, when they 'behave' green. This 'cost' will be the difference of the economic utility functions[5]:

$$\Delta U_e = f(n_2) - f(n_1)$$

As usual, we have that $f(\cdot)$ has to be an increasing function with diminishing marginal returns. At this point, I will use two different utility functions, which will have different effects. As with the moral satisfaction function, β will represent the weight of the economic part in the overall utility function of the agent:

$$f_1(n) = \beta \ln(n) \tag{2.7}$$

$$f_2(n) = \beta\sqrt{n} \tag{2.8}$$

Replacing the previous functions in ΔU_e, we get:

$$\begin{aligned}
\Delta U_e^1 &= \beta \ln(n_2) - \beta \ln(n_1) \\
&= \beta(\ln(n_2) - \ln(n_1)) \\
&= \beta \ln\left(\frac{n_2}{n_1}\right) \\
&= \beta \ln\left(\frac{(1-\delta)w}{w}\right) \\
\Delta U_e^1 &= \beta \ln(1-\delta)
\end{aligned} \tag{2.9}$$

[4]The biased perception could be due to the results of different effects, as for example: publicizing the effort made by some 'famous' people ($\Omega > 1$) or, on the contrary, by reports of how people do not care of environment ($\Omega < 1$).

[5]Recalling equations 2.1 and 2.2

$$
\begin{aligned}
\Delta U_e^2 &= \beta\sqrt{n_2} - \beta\sqrt{n_1} \\
&= \beta(\sqrt{n_2} - \sqrt{n_1}) \\
&= \beta\left(\sqrt{(1-\delta)w} - \sqrt{w}\right) \\
\Delta U_e^2 &= \beta\sqrt{w}\left(\sqrt{(1-\delta)} - 1\right) \qquad (2.10)
\end{aligned}
$$

As it can be seen from the previous two equations, the choice of the utility function can have some interesting impacts (to be seen in the equilibria analysis). In the meantime, we can see that in equation 2.9 the value of ΔU_e^1 is independent of the wage level w, whereas in equation 2.10 the difference ΔU_e^2 is actually increasing with w, proportional to $\sqrt{w}$.

This means that, in the latter case, the 'cost' of being green increases with the level of wage. Analysing if there is a 'sound' utility function which could have the inverse effect, we can see that if we choose $f(n) = \ln(n+K)$, where K is a fixed positive constant ($K > 0$):

$$
\begin{aligned}
\frac{\partial \Delta U_e}{\partial w} &= (1-\delta)f'((1-\delta)w) - f'(w) \\
\frac{\partial \Delta U_e}{\partial w} &= \frac{(1-\delta)}{(1-\delta)w+K} - \frac{1}{w+K} \\
\frac{\partial \Delta U_e}{\partial w} &= \frac{-\delta K}{((1-\delta)w+K)(w+K)} < 0
\end{aligned}
$$

Even though $\frac{\partial \Delta U_e}{\partial w} < 0$, we can see that if w grows, then $\frac{\partial \Delta U_e}{\partial w} \to 0$. The other possibility is to choose an utility function with a ceiling, as for example $f(n) = \beta\frac{(n-1)}{n}$. Therefore $\Delta U_e^3 = \beta(\frac{n_2-1}{n_2} - \frac{n_1-1}{n_1}) = -\beta\delta w/(1-\delta)$. In that case, we find that the value of ΔU_e^3 decreases as w increases. In the subsequent pages, I will use $f(n) = \ln(n)$, unless another thing is stated.

3 The evolution of Society

Now we will focus on how society evolves. In this work, 'evolves' is to be interpreted as the proportion of green people (and conversely brown people) that changes from period to period. I will use the concept known as 'replicator dynamics' from Taylor and Jonker (1978) [12]. The replicator dynamics says that the growth rate of a strategy (in our setting: to be green) is proportional to the success of that strategy, where "success" is measured by the moral payoff minus its 'costs'.

Using the replicator dynamics motion equation, we have:

$$\Delta q_{t+1} = q_{t+1} - q_t = q_t(1 - q_t)(m(r,q) + \Delta U_e) \tag{3.1}$$

where $m(r,q)$ is the moral gain and ΔU_e (which is always negative) is the constant cost (with respect to q) of being green.

As can be seen from the motion equation, there are at least two equilibrium points: $q^* = 0$ and $q^* = 1$, plus the one(s) given by the third parentheses. Since $(m(r,q) + \Delta U_e)$[6] is positive when $q_t \approx 0$ and it is negative when $q_t \approx 1$, then these two points are unstable.

Replacing the functions $m(r,q)$ and ΔU_e with equations 2.6 and 2.9 respectively, we get:

$$\Delta q_{t+1} = q_t(1 - q_t)(\alpha\gamma r(1 - q)(1 - rq) + \beta \ln(1 - \delta)) \tag{3.2}$$

Finally, using the previous functions and some 'sound' parameters[7], we can have two possible outcomes: A green equilibrium ($q^* > 1/2$) or a brown one ($q^* < 1/2$). These equilibria can be observed in the following figures 1 and 2, where we have that:

Curve	Meaning
$m(r,q)$	**Green**: The moral gain or satisfaction at a level q of green population proportion.
ΔU_e	**Red**: The cost, in utility units, of being green. It does not depend on q.
$m(r,q) + \Delta U_e$	**Blue**: The sum of the two previous curves. It is the total gain (or loss) of being green.
Δq_{t+1}	***Dotted line***: The result of the motion equation. A positive value means that q will be grow in the next period, where a negative value means a decline of q.
q_{t+1}	**Black:** The value of q_{t+1} as a function of q_t

The fact that ΔU_e is independent of q_t is quite obvious, since ΔU_e has to do with the level of green willingness δ and the relative prices between brown and green goods, represented by ρ. Given these two parameters, and possibly the level of income w, the value of ΔU_e is defined, which does not have relationship with q_t.

[6] To be checked in the following pages, assuming some 'sound' parameters.

[7] The parameters were chosen as follows: δ at a 10% level, which might seem a little high, although changing the value, towards 5% for example, does not change the results. The green mark up ρ between 20% and 30%, which seems reasonable for an average green mark up. α and γ, which get combined together, result in being the measure of the moral gain versus β which is the weight of the economic gain. It seems reasonable for β to be higher than $\alpha\gamma$, since the U_e has diminishing marginal returns, whereas the former does not. Nevertheless, the last three parameters have been chosen in order to have the equilibria around $q^* = 1/2$, in order to ease the analysis.

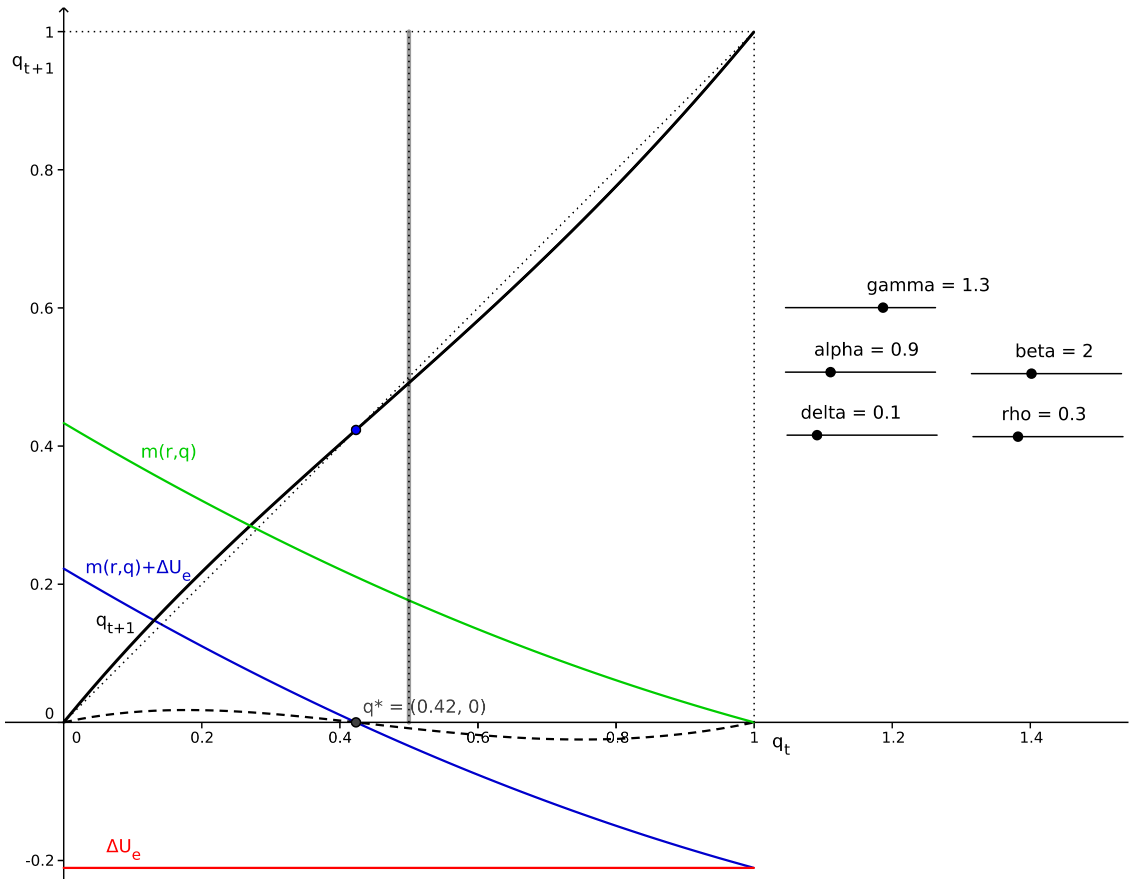

Figure 1: Case with *brown* equilibrium

Now looking at $m(r,q)$, we can see that it is a decreasing function of q_t. This also makes sense, since the moral satisfaction comes from the personal level of 'green help' r compared to the average level of the same concept, $\bar{r}$. Since the mean value $\bar{r}$ will increase with q_t (for a constant r) then the resulting moral satisfaction will decline with q_t, and therefore the decreasing function $m(r,q)$.

Recalling the concept of the *'distorting factor of perception'* Ω from section 2.4, it is interesting to see that the equilibrium point will change with Ω. If $\Omega > 1$, the perceived $\bar{r}$ will be higher and therefore the equilibrium point will move to the left, becoming a more brown equilibrium, compared to the base case. As expected, the converse is also true, having that a value of $\Omega < 1$ will move the equilibrium point to the right, to a greener equilibrium. The idea here, as it is discussed in Nyborg et al.(2003) [10] and Brekke et al.(2003) [4], is that when $\bar{r}$ seems to be higher, people might think that the rest of the society *has already taken over the responsibility* of environment conservation and therefore their moral gain will decline.

As was mentioned at the beginning of the present section, we can now verify that when $q = 0$ or $q = 1$ the equilibria are actually unstable. This can be easily seen from the fact that $m(r,q) + \Delta U_e$

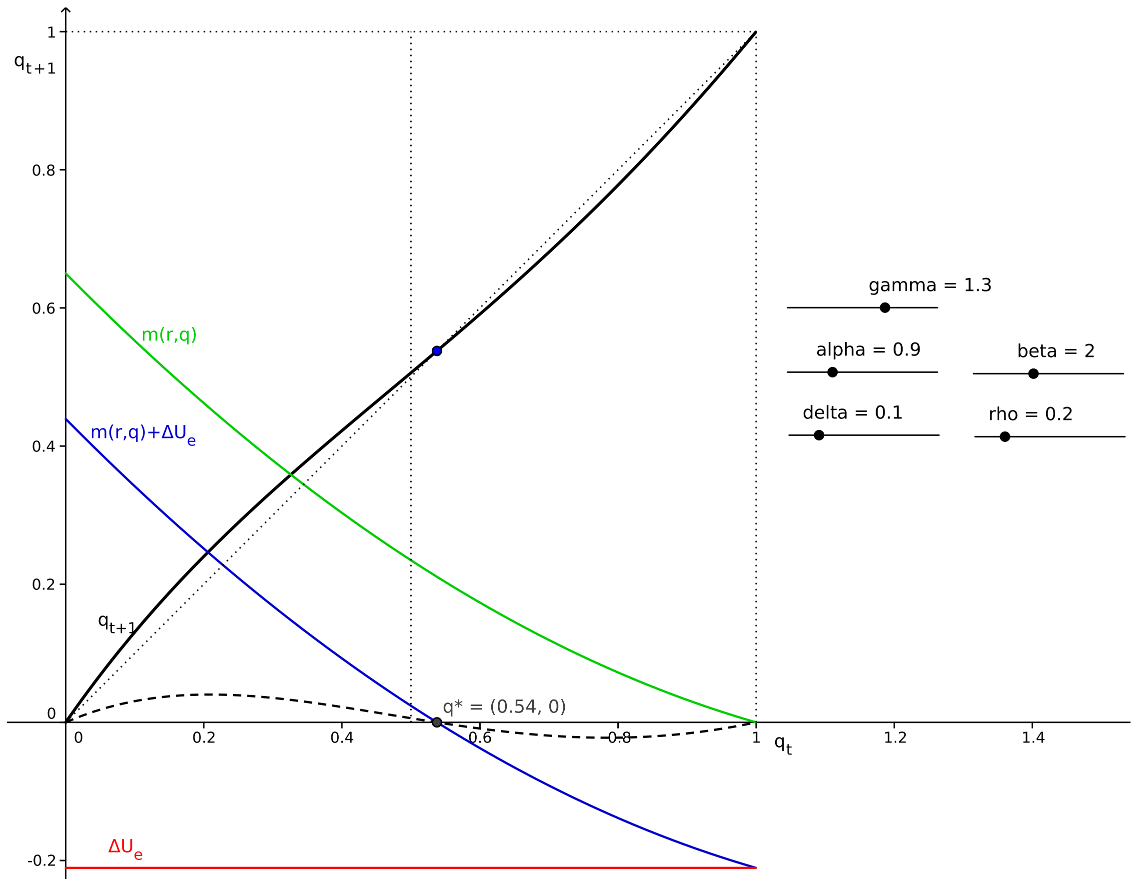

Figure 2: Case with *green* equilibrium

is positive when $q = 0$ and negative when $q = 1$.

4 The equilibrium point q^*

Now I will calculate the equilibrium point q^* and then see how a green tax/subsidy can impact on it. Recalling the motion equation of q_t from section 3, equation 3.2, we have:

$$\Delta q_{t+1} = q_t(1 - q_t)(\alpha\gamma r(1 - q)(1 - rq) + \beta \ln(1 - \delta))$$

Therefore, setting the last term to zero, we have that the equilibrium point q^* must satisfy:

$$\alpha\gamma r(1 - q)(1 - rq) + \beta \ln(1 - \delta) = 0 \tag{4.1}$$

or

$$\alpha\gamma r(1 - q)(1 - rq) = -\beta \ln(1 - \delta)$$

Knowing the fact that, for small values of δ, we have that $\ln(1-\delta) \approx -\delta$ and replacing r with its value, we get the following second degree equation for q^*:

$$\delta q^2 - ((1-\delta)\rho + \delta)q + \frac{(1-\delta)\rho}{\alpha\gamma}\Big(\alpha\gamma - (1-\delta)\rho\beta\Big) \approx 0 \tag{4.2}$$

which yileds to:

$$q^* \approx \frac{1}{2} + \frac{(1-\delta)\rho}{2\delta} \pm \frac{\sqrt{((1-\delta)\rho+\delta)^2 - \frac{4\delta(1-\delta)\rho(\alpha\gamma-(1-\delta)\rho\beta)}{\alpha\gamma}}}{2\delta} \tag{4.3}$$

Although we get a the 'complex' form for the equilibrium point, we can deduce some interesting things from it. First, we will see that one of the two roots of q^* is actually greater than 1, therefore leaving us only to deal with the second one.

For the bigger root to be less or equal to 1, we must have:

$$\frac{1}{2} + \frac{(1-\delta)\rho}{2\delta} + \frac{\sqrt{((1-\delta)\rho+\delta)^2 - \frac{4\delta(1-\delta)\rho(\alpha\gamma-(1-\delta)\rho\beta)}{\alpha\gamma}}}{2\delta} \leq 1$$

which would imply that:

$$\frac{(1-\delta)\rho}{2\delta} + \frac{\sqrt{((1-\delta)\rho+\delta)^2 - \frac{4\delta(1-\delta)\rho(\alpha\gamma-(1-\delta)\rho\beta)}{\alpha\gamma}}}{2\delta} \leq \frac{1}{2}$$

$$(1-\delta)\rho + \sqrt{((1-\delta)\rho+\delta)^2 - \frac{4\delta(1-\delta)\rho(\alpha\gamma-(1-\delta)\rho\beta)}{\alpha\gamma}} \leq \delta$$

Since the square root has to be greater or equal to zero and recalling that $\frac{\delta}{(1-\delta)} = r\rho$, we have:

$$\begin{aligned} (1-\delta)\rho &\leq \delta \\ \rho &\leq \frac{\delta}{(1-\delta)} \\ \rho &\leq r\rho \\ 1 &\leq r \end{aligned}$$

Which would mean that $q^* = 1$, $r^* = 1$ and that the value inside the square root is also equal to zero. I will assume that these extremely specific conditions (regarding α, β, γ and ρ) do not hold and, therefore, we are left with only one solution for q^*:

$$q^* \approx \frac{1}{2} + \frac{(1-\delta)\rho}{2\delta} - \frac{\sqrt{((1-\delta)\rho+\delta)^2 - \frac{4\delta(1-\delta)\rho(\alpha\gamma-(1-\delta)\rho\beta)}{\alpha\gamma}}}{2\delta} \tag{4.4}$$

So now we can study whether the equilibrium point is a brown or green one. Following a similar reasoning from the previous one, we find that, to get a green equilibrium, we need:

$$\frac{1}{2}+\frac{(1-\delta)\rho}{2\delta}-\frac{\sqrt{((1-\delta)\rho+\delta)^2-\frac{4\delta(1-\delta)\rho(\alpha\gamma-(1-\delta)\rho\beta)}{\alpha\gamma}}}{2\delta}\geq\frac{1}{2}$$

which gives us the following condition for a green equilibrium:

$$2(1-\delta)\rho>\delta+\frac{4(1-\delta)^2\rho^2\beta}{\alpha\gamma} \tag{4.5}$$

And rearranging for ρ:

$$4(1-\delta)^2\beta\rho^2-2\alpha\gamma(1-\delta)\rho+\alpha\gamma\delta<0$$

which finally gives us ρ^*, which much lie between these two values:

$$\rho^*=\frac{\alpha\gamma\pm\sqrt{\alpha^2\gamma^2-4\alpha\beta\gamma\delta}}{4\beta(1-\delta)} \tag{4.6}$$

It is important to notice that for a green equilibrium to exist (a real solution of ρ), we need for the discriminant to be greater or equal to zero, giving us the following condition:

$$\alpha\gamma\geq 4\beta\delta \tag{4.7}$$

Now we will go back to the analysis of the solution of q^* and how it changes with δ and ρ. Due to the complexity of the formulation (given in equation 4.4), I will rely on simulation. For this matter, I will use the previous values of α, β and γ, as shown in figure 3. Nevertheless and observing previous equation 4.4, we can see that the effect of α (and in the same manner γ) will be a shift up and down of the curve (with some deformations). In the same fashion, but with opposite direction, will be the effect of β on the curve $q^*(\delta)$.

It is important to notice that depending on the values taken by δ and ρ, r will reach its maximum value of 1. For example, let's check for the case of $\rho = 0.3$ (Figure 3). As δ grows, so does its corresponding value of r (dotted orange line), reaching its maximum value, shown with the dotted vertical red line. From this point on, $r = 1$ (for this particular value of ρ) and therefore the change in the shape of the curve (kink). For value of $\rho \geq 0.45$ this kink becomes much more evident. The idea here is that when r reaches its maximum, any new green 'effort', which is an increase in δ, only adds more 'cost' to the green agent, but no more satisfaction, making q^* decrease (even though it was increasing at that point, before reaching $r = 1$).

One interesting thing to point out is the fact that when γ decreases, q^* will decrease too. This causality could have some curious effect. For example, let's suppose that we are in a green equilibrium ($q^* > 1/2$) and, due to an external cause (such as a technological improvement), γ decreases,

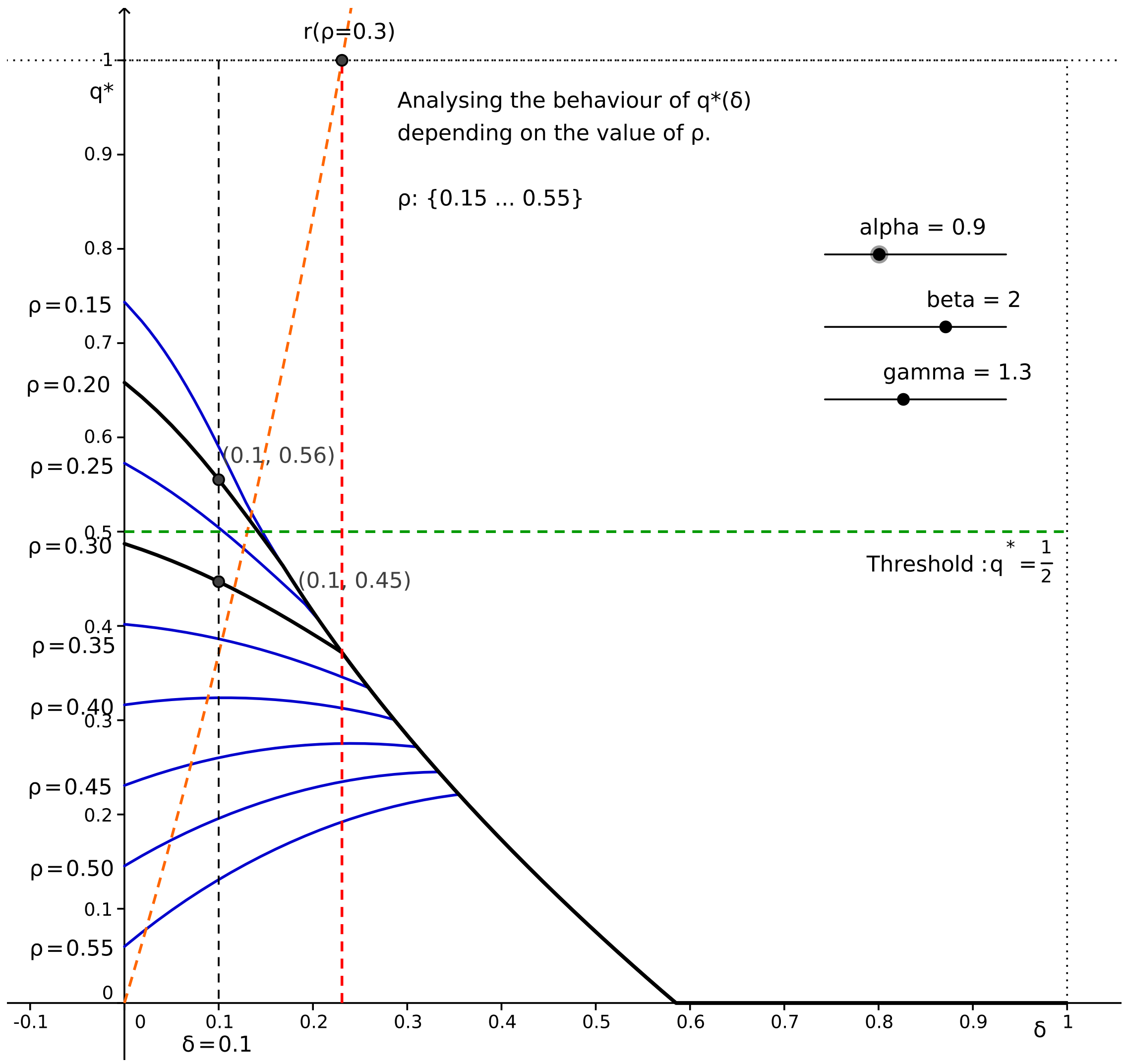

Figure 3: Analysis q*

meaning that the brown products now pollutes less. In principle, this should be 'good' news. However, we will also end up with a lower equilibrium, maybe now in a brown point ($q^* < 1/2$), therefore maybe having a counter effect.

Also two cases have been highlighted: $\rho = 0.2$, giving a green equilibrium and $\rho = 0.3$, leading to a brown one. These two cases were also analysed in previous figures 1 and 2 and are highlighted here for comparison matters.

For high values of ρ, the mark up of green products, we find that the equilibrium point is increasing with δ, the green willingness (lower curves of the graph). But, in this case (high ρ) we always end up with a brown equilibrium. On the contrary, for 'milder' values of ρ, we get the opposite effect, meaning that the equilibrium point moves to a more brown state, for a higher value of δ, the green willingness.

The intuition here is that when the mark up for green products is high ($\rho \approx 0.5$), we will be in a range of q^* which will be low (brown). We will also have a low value of r, the green proportion of green people. Therefore, an increase in δ, which will translate in an certain increase in r, will have a high impact in the moral gain (since q is low[8]). This increase in moral gain will overcome the 'lost' in utility units due to the increase in δ , and therefore will make q^* grow.

On the contrary, when the mark up is in a lower level ($\rho \approx 0.25$), an increase in δ will have a different impact. Even though the same increase in δ will imply a greater increase in r (with respect to the previous case), a lower level of ρ will have us in a higher equilibrium point q^*. The later will translate as follows: the increase in r will not be followed by the corresponding increase in the moral benefit, since the fact that q has a higher value (see footnote). Therefore, the increase in moral gain will be completely thwarted by the given 'cost' due to the initial increase in δ, making q^* drop.

Now focusing on the changes due to ρ, we can see that as ρ decreases, the equilibrium point q^* will increase, and depending on the value of δ could actually cross the brown/green government threshold.

5 The green government

In the following section I will introduce the difference between having a brown or a green government. As was explained in the introduction and in the first paragraphs of section 2, the type of government elected will respond to a simple majority rule, meaning that if $q_t < 1/2$ we will have a brown government, where if $q_t \geq 1/2$ we will get a green one.

In the first case, the government will just leave the market work, i.e. it will not change the prices of the brown or green goods, nor implement any kind of incentive to pollute less. It is just not in the best direct interests of its constituents, the brown people[9].

But in the second instance, the government will actually make some changes. In this model, when in the presence of a green government, it will be introduced a lump sum tax τ on their income, to everybody. Therefore the available income will change from w to $(1-\tau)w$. With this tax, the government will subsidy the green goods, changing it price from $(1+\rho)$ to $(1+\rho-\epsilon)$,

[8]It might be useful to recall that $\Delta U = \alpha\gamma r(1-q)(1-rq) + \beta \ln(1-\delta)$

[9]From a direct economic point of view.

where ϵ is the value of the subsidy[10].

5.1 Subsidising the green products

The idea here is that the government will levy a tax, with a rate τ on the income, in order to subsidy the green good. I will call ρ' the new value of the mark up of the green good, where $\rho' = \rho - \epsilon$. With this, we will get a new value of r, which will be called r'. Obviously the government has no intention to overcharge the population with an excessive tax rate, therefore we have two direct constraints:

$$\begin{aligned} \rho' &\geq 0 \\ r' &\leq 1 \end{aligned}$$

Meaning that we neither want to make green products cheaper than the brown ones (the same price will actually be enough), nor have a proportion (of green buying among the green people) higher than one. We also will assume that the government budget constraint is level, therefore all the money collected in taxes is used in the same period for subsidies. Then,

$$\begin{aligned} \textit{Total subsidy} &= \textit{Total tax collected} \\ \epsilon r' q n_2' &= \tau w \\ \epsilon &= \frac{\tau w}{r' q n_2'} \end{aligned} \tag{5.1}$$

We also have the new (total) amount of goods bought by each type of person is now:

$$\begin{aligned} n_1' &= (1-\tau)w \\ n_2' &= \frac{(1-\tau)w}{1+r'\rho'} \end{aligned}$$

So, replacing $\rho' = \rho - \epsilon$ on 5.1 and doing some math, we get:

$$r' = r + \frac{\tau}{q\rho(1-\tau)(1-\delta)} \tag{5.2}$$

As can been seen in the relationship 5.2, $r' > r$, which was something one would expect. The difference among them is increasing with τ (as expected) and decreasing with q and ρ. The first relationship, of the last two, is also quite intuitive. The fewer green people in society, the stronger the effect of the tax on the price and therefore the increase of r, since the tax is levied on all the population. The second effect, coming from ρ, has to do with the fact that if the green price is too

[10]Some other options could be added to the model, as for example a method to change the value of γ. Mechanisms such as these are not considered, since they get out of the scope of the present model, although it could be thought as extensions of the present work.

close to the brown one ($\rho \approx 0$), then a small subsidy will make all green people consume only green products. As always, we have to remind that $r' \leq 1$.

Replacing r on 5.2 and using the last inequality ($r' \leq 1$), we have:

$$r' = r + \frac{\tau}{q\rho(1-\tau)(1-\delta)} \tag{5.3}$$

$$\tau \leq \frac{(\rho(1-\delta)-\delta)q}{1+(\rho(1-\delta)-\delta)q} \tag{5.4}$$

In other words, the government will not use a green tax higher than what is stated in equation 5.4, due to the arguments previously explained.

In order to know how q^* moves with respect to r (or δ), I will analyse the derivative of q^* with respect to r. Observing the figure 4, we can see that for 'usual' range of r ($r \leq 0.84$), the derivative is always positive. This means that a rise in r will actually increase the equilibrium point. This is important due to the fact that when we are changing from a brown into a green government, we want to see if this transition will remain, or if it will turn out unstable (due to the fact that the green government implements the green tax and therefore increases r). Since the derivative is positive, we can conclude that the transition is actually 'stable'.

In the example below, we have $\rho = 0.3$, which gives us a corresponding value of $r = 0.37$, watching the dotted green curve (with $\delta = 0.1$ as usual). At this level of r, we can see that q^* will almost grow as much as r.

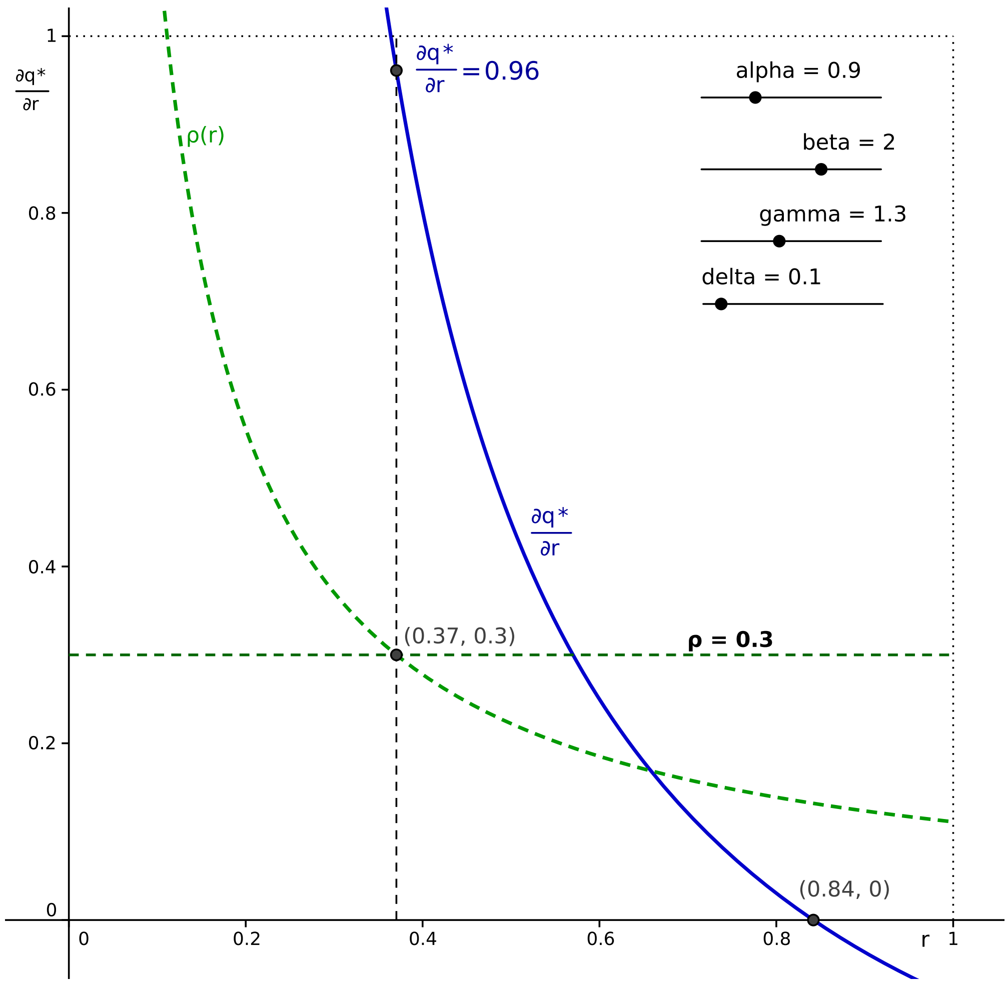

Figure 4: Analysing q* with respect to r

6 Equilibrium Points and Total Welfare

Now that we have the different relationships between the variables and possible equilibria (brown or green), we will first analyse what would happen if we were in a green equilibrium. In order to do this, I will rely in the following figure 5. Hoping that the graph is not too tangled to the reader, I will start describing its construction.

To start with, I would like to remind that the tax rate τ has a limit, which was seen in the conditions stated in equation 5.4. Since this constraint depends on q, it was plotted in order to verify that we are actually moving on the allowed areas of the graph. In this case, at the selected value of $q = 0.56$, the maximal value of τ would be 0.09, which is much bigger than the actual one of 0.03 used.

Given this, we can now notice that we have two curves $r'(q)$ (the black ones). The 'first' one (the decreasing one), is given by equation 5.2, which has to do with the effect of τ, the tax rate, on r'. The 'second' one is the inverse function of the solution of q^*, as in equation 4.4. The point where they meet [0.56 , 0.57] is the solution for that given value of τ. In this case we got $q^* = 0.56$ and $r' = 0.57$.

Now, by changing the value of τ, we will shift the first curve to the right. This will move the intersection point to the right (not too much) and mostly upwards. This means that the 'effort' on augmenting τ (at this level) mainly translates into the green people consuming more green products, but not in a bigger change in the society composition.

Now let's observe the different welfares. First, I will focus on the 'apparent' welfare, that is, the agent's welfare without the pollution disutility. These two curves are represented by the brown and green dotted lines on the graph[11]. As we can see, the brown 'apparent' utility is constant in each type of government, having a lower value in the green government, due to the payment of the tax rate τ. The green one, which has the same 'jump' in $q = 1/2$, is a decreasing one, as we saw previously in figures 1 and 2.

Therefore, if the green government chooses a tax level τ in order to stay in office, it will choose a τ such as q is just greater than $1/2$.

Now, let's focus on the pollution disutility. As we saw in the beginning of this work, we called $h(P_{t-1})$ the disutility function for certain pollution level P_{t-1}. We know that this disutility function is increasing with the level of pollution (which could be, for example, in a linear or exponential way). We will just focus on a linear form (with a constant factor of ϕ). Since we are analysing the

[11]A 'zoom' factor has been used in order to place all the curves in the same range of the *y axis*.

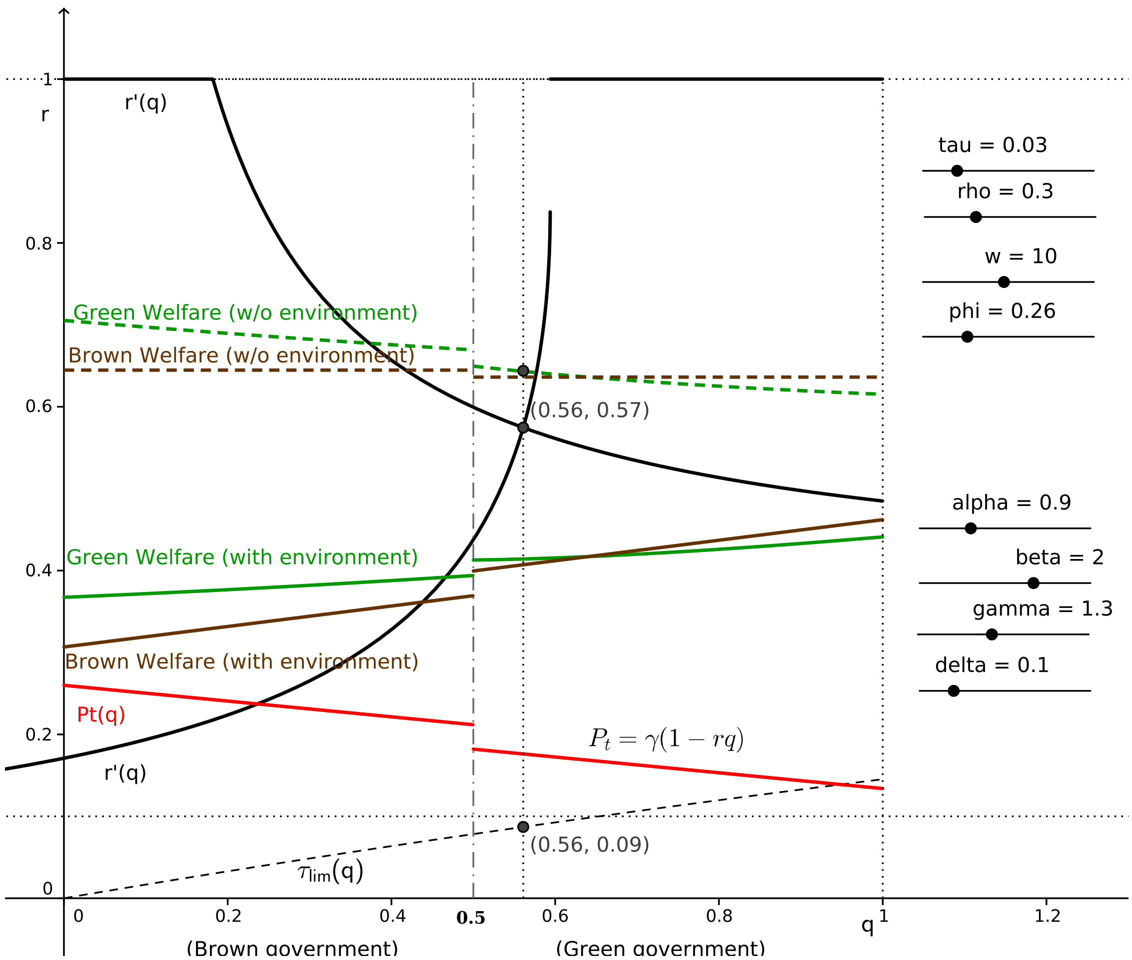

Figure 5: Analysing τ and welfares

equilibrium states, then $P_t = P_{t-1}$. Recalling that $P_t = \gamma(1 - rq)$, we can plot the pollution level in our graph, which is represented by the red curve. Therefore, as the green population grows, the pollution decreases and, with it, its disutility, adding up to a higher utility when $q \to 1$[12].

Hence, returning to our green government choice of the tax rate τ, now if it actually incorporates the pollution effect upon the green people (or for this matter, on everybody's welfare), the government would like to have a higher tax rate. This will be the case until the intersection point goes all the way up to $r = 1$ (around $q^* = 0.6$), since there is no gain to overtaxing after this point. In the present figure, I have chosen a value of $\tau = 0.03$. This has been done with two simple academic objectives. First, to allow the graph to be in a simpler point to study it; and, secondly, to have an intermediate point (between the two previous assumptions).

Although the Total Welfare was not graphed in figure 5 (to avoid the graph becoming any

[12]For simplicity, I have assume that ϕ is high enough to have this effect. It could be the case that ϕ is too low and the overall welfare still decreases with q.

more complex) we can make similar deductions to the ones made earlier. Since the Total Welfare should 'treat' brown and green people in the same way, we consequently would have their utility functions (per capita) be the same weight. Therefore, the total welfare (TW) would be:

$$TW = q \cdot U_{green} + (1-q) \cdot U_{brown} \tag{6.1}$$

In other words, the Total Welfare is a weighed average (proportional to the share of each type) of each specific welfare. Looking again at the figure, we can see that both curves (brown and green ones) move in the same fashion, and, therefore, conserve the previously analysed behaviour.

7 Returning to the dymanics

Last but not least, we return to see how our final evolution curve has been shaped. As it can be seen in figure 6, we now have a curve that has a jump in $q = 1/2$. I have used the same values as before (including the tax rate τ), and the same curves and color as in figures 1 and 2. It highlights the two equilibria points q_1^* and q_2^*, that is, the brown and green equilibria respectively.

As can be clearly observed from the graph, both points are stable. Moreover, we have a 'pollution trap' effect, meaning that the result is dependent on the initial value of q. If we start from a value of $q_0 < 1/2$, we will end up in the brown equilibrium, whereas if we start in a point $q_0 > 1/2$, we will end in the second equilibrium point q_2^*, the green one.

At this point, some comments can be made with respect to the possible impact of the income level w on the previous result. So far, I have assumed that the economic utility function $f(n)$ has a logarithmic shape, given that the result ΔU_e does not depend on the level of w. However, if that were not the case[13], then the wage level will have an impact on the equilibria. Shifting up and down the (red) curve ΔU_e, which conversely will do the same with the (blue) curve $m(r,q) + \Delta U_e$. This will move our equilibria points to the left or right. This could give us only one type of equilibrium: a brown or green one.

This last point has important significance, since it could explain why there are, at a given point, countries with high income like the United States which pollute more than others with a lower income, who turn out to be much more environmentally friendly[14]. Obviously, this is not the only effect that can actually shift the curves and therefore the equilibria points. The personal green satisfaction intensity could have a cultural background. Therefore changing it (α in our model) will have a similar impact, giving us again different equilibria results for two, initially, similar

[13]For example, a function $f(n) = \sqrt{n}$ will give us a increasing value of ΔU_e with respect to w, whereas a function $f(n) = \frac{(x-1)}{x}$ will give us the opposite effect.

[14]Information about this fact can be gathered from the International Energy Agency [13] and the World Bank's CO2 emissions [14], to mention a few.

countries (as with the U.S. and the Nordic countries, which have similar GDP per capita).

Finally, using the present model as a starting point, one could study and check the possibility of public policy (or private green driven ones) switching from a brown 'pollution trap' to a green one. For example, as stated in section 3, an eventual change in the perception (Ω) of the 'mean' contribution ($\bar{r}$) could also move the curves and therefore the consequent result.

This could eventually give some possible extensions of the present work. Besides the ones already mentioned above, one could analyse the impact of the existence of a variety of green willingness degrees in the 'green willingness'. The idea here, similar to the one used in Nyborg et al.(2003) [10], is to have not only one type of people with willingness δ, but to have different groups with different 'green willingnesses'. Hence, we could have different values for δ: $\delta_1 > \delta_2 > \cdots > \delta_n$. This could modify the evolution motion equation, and therefore the possible equilibria, as well as the impact of the different parameters, eventually showing some new interesting features.

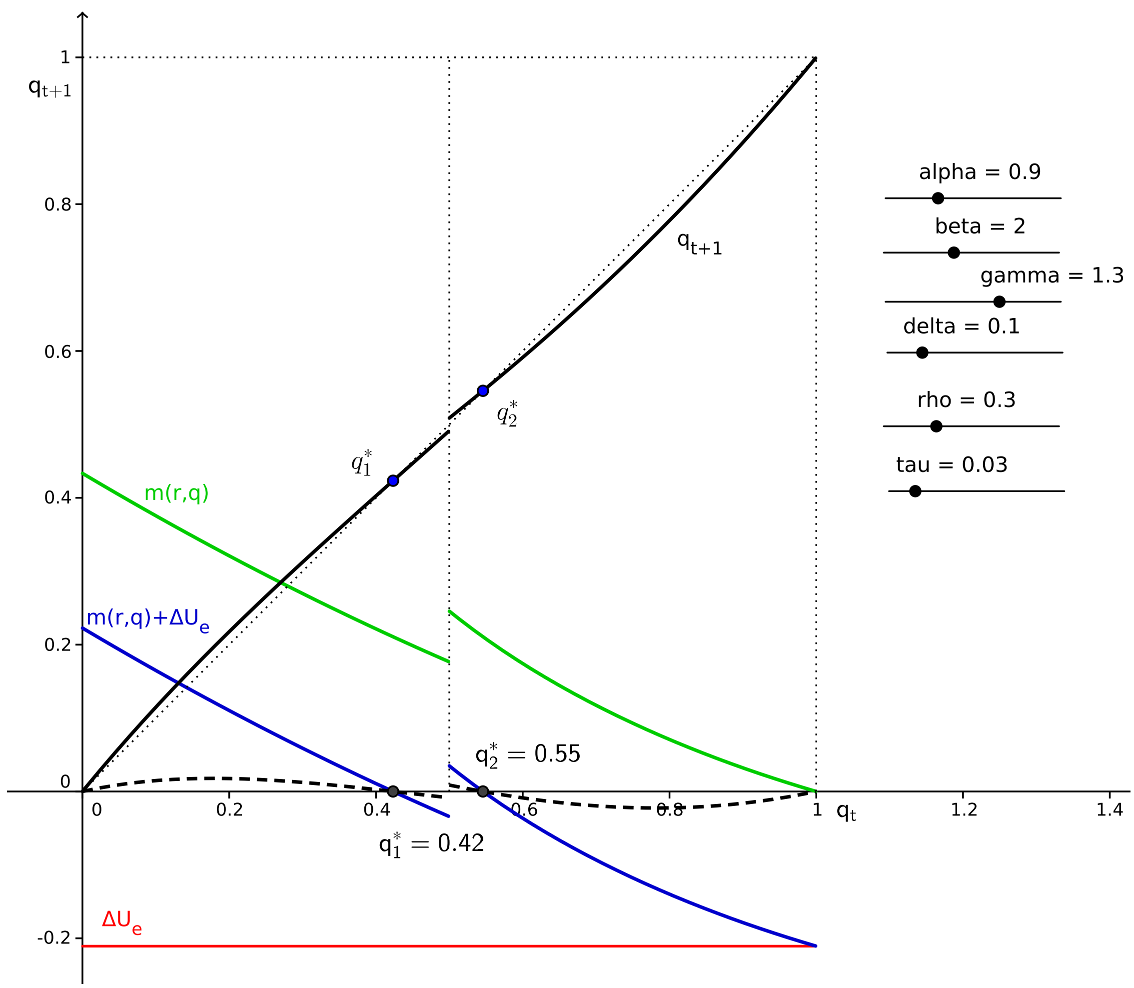

Figure 6: Final Dymanics

8 Conclusions

We have seen a model of evolution with some type of preferences, in this case to be 'green', where 'success' or 'failure' (in an evolutionary way) has to do with moral satisfaction, as in Nyborg et al.(2003) [10]. Therefore the number of green people will increase if being green has a positive pay-off, and the contrary if not. Taking this natural idea into consideration and translating this into a green/brown consuming behaviour, we have been able to analyse how the environment and society could evolve.

The main idea here has been to check if attaining a sustainable (green) environment is politically possible. In my model, 'politically' just means checking if there is a sufficient number of green people in order to move forward with green policies. In this case the policy has been the implementation of a lump sum tax (imposed on the whole population) in order to subsidise the consumption proportion of the green product.

In doing so, I have been able to study the behaviour of the evolution of this society and to find some equilibria points, i.e. where the proportion of green and brown people stays the same. It has been found that, according to some sound assumptions, there could exist two such points: a brown equilibrium and a green one. This means that we can end up with different type of societies, even though they have the same original characteristics, and are only differentiated by their starting point. (the initial proportion of green and brown people).

Having that type of result could actually explain some known results. For example, the fact that two (theoretically) similar countries could actually have quite different behaviours with respect to the treatment of pollution (as with the U.S. and Nordic countries). This type of model can also explain why countries with low income do actually care about the environment, even in some cases more that their developed counterparts.

Due to the last facts, it seems possible to further develop this idea in order to fine tune the concepts presented here and subsequently check them with some real data. Studies concerning the linkage between the moral (green) satisfaction and the final outcome of each country, could be the next step to follow. Discovering these facts could lead to some interesting and useful results in order to promote and attain a better environment. After this, it could be possible to analyse the interaction of different countries (in the way they evolve, in the terms used in the present work) and come up with further insights.

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